Principal-Agent Games of Optimal Stopping

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More precisely:

- Irreversible investment under uncertainty and with competition
  $\longrightarrow$ **Game of Optimal Stopping**

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Transition from internal combustion engine vehicles to battery-electric vehicles.

More precisely:

- Irreversible investment under uncertainty and with competition
  $\longrightarrow$ **Game of Optimal Stopping**
- Socially optimal policy design and cost-benefit analysis
  $\longrightarrow$ **Principal-Agent Problem** with **Asymmetric Information**

Economic Background

European Green Deal:

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The Irreversible Investment Problem

Examplary Model of Consumer Preferences: Wright-Fisher Diffusion

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- This is just an example an can be generalized significantly...

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$\longrightarrow$ Much more on this in **Emanuel's talk**!

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Let $u_1, u_2 : [0,1] \times \{0,1\} \to \mathbb{R}$ and let $D_1(1), D_1(0), D_2(1), D_2(0)$ be proactive subsets of $[0,1]$. Suppose that, for $i, j \in \{1,2\}$ with $i \neq j$, the following properties hold:

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Then the stopping times associated with $D_1(1), D_1(0), D_2(1), D_2(0)$ form a Nash equilibrium.

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Suppose that $D_1(1), D_1(0), D_2(1), D_2(0)$ are proactive subsets of $[0,1]$ such that the associated stopping times form a Nash equilibrium. Denote the corresponding value functions by $v_1, v_2 : [0,1] \times \{0,1\} \to \mathbb{R}$. Then, for $i, j \in \{1,2\}$ with $i \neq j$, the following properties hold:

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Time for some sketches!

The Principal-Agent Problem

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- Influences the reward structure of the agents and dynamics of X:

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- How to quantify response of the agents to a policy in the absence of uniqueness of Nash equilibria?

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- The policymaker's reward signal associated with each $(\tau_1^*, \tau_2^*) \in T(\alpha)$ takes the form

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- Otherwise, one has to replace the expectation in J_0 by a more general preference relation (maybe happening eventually...).

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Optimal Policies

The mapping

$$\alpha \mapsto J_0\big(x, z, \alpha, \tau_1^\alpha, \tau_2^\alpha\big)$$

is continuous and hence there exists an optimal $\alpha^* = \alpha^*(x, z) \in \mathbb{A}$.

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- Numerical experiments with realistic data seem to indicate that first-movers are usually the largest agents or risk-seeking newcomers.
- Results appear to be quite sensitive to the choice of discount factors.
- Plethora of different effects, especially due to the non-uniqueness of equilibria. We still have a lot to explore...

Conclusion:

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Thanks for your attention!