Principal-Agent Games of Optimal Stopping

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Joint Work in Progress with **Emanuel Rapsch** (TU Berlin).

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- Irreversible investment under uncertainty and with competition
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Transition from internal combustion engine vehicles to battery-electric vehicles.

More precisely:

- ▷ Irreversible investment under uncertainty and with competition
  $\longrightarrow$ **Game of Optimal Stopping**
- ▷ Optimal policy design to accelerate the transition
  $\longrightarrow$ **Principal-Agent Problem** with **Asymmetric Information**

Economic Background

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- But: Consumer demand will grow with time

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The Irreversible Investment Problem

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⟶ Instead of pairs of stopping times (τ_1, τ_2), our controls are $D_1(z)$ and $D_2(z)$ for $z \in \{0, 1\}$, where

$$D_1(0), D_1(1), D_2(0), D_2(1) \qquad \text{are closed subsets of} \qquad [0, 1].$$

Characterization of Equilibria I

Suppose that $D_1(z), D_2(z)$ are optimal and denote the corresponding value functions by v_1, v_2. Then, for $i, j \in \{1, 2\}$ with $i \neq j$, it holds that

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Then D_1, D_2 are optimal.

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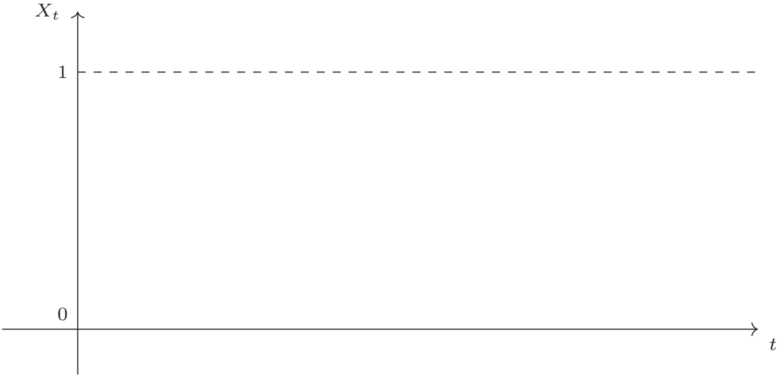

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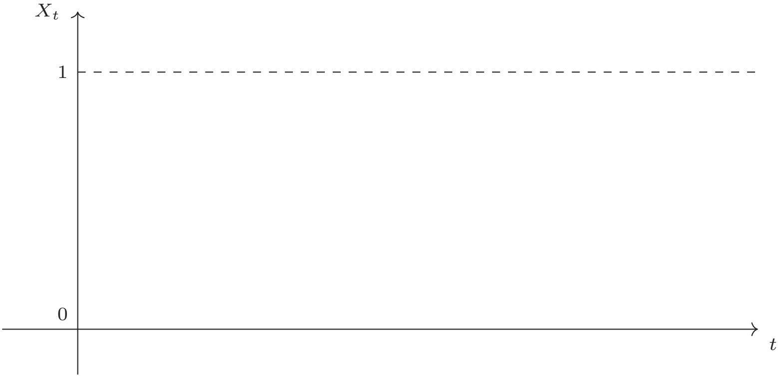

The Principal-Agent Problem

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- How to quantify response of the agents to a policy in the absence of uniqueness of Nash equilibria?

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- Otherwise, one has to replace the expectation in J_0 by a more general preference relation.

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Optimal Policies

The mapping

$$\alpha \mapsto J_0\big(x, z_1, z_2, \alpha, \tau_1^\alpha, \tau_2^\alpha\big)$$

is continuous and hence there exists an optimal $\alpha^* = \alpha^*(x, z_1, z_2) \in \mathbb{A}$.

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- Numerical experiments with realistic data seem to indicate that first-movers are usually the largest agents or risk-seeking newcomers.
- Results appear to be quite sensitive to the choice of discount factors.
- Plethora of different effects, especially due to the non-uniqueness of equilibria.

Conclusion:

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Thanks and Good Night!